Chapter 3 Factors and Products – Notes

◆<u>3.1 Factors and Multiples of Whole Numbers (p. 134)</u>

- <u>Prime</u> and <u>Composite Numbers</u> - Vocabulary

→***Factors** of a number:* numbers that are <u>multiplied</u> together to get another number (the product)

→***Product:*** the number that results when two or more factors are multiplied.

Example: 2 x 3 = 6; the 2 and 3 are <u>factors</u> of 6, while 6 is the <u>product</u>.

You can arrive at the <u>product of 16</u> by **multiplying** the following factors (numbers):

1 x 16 = 16 or 2 x 8 = 16 or 4 x 4 or 4^2 = 16

Therefore, the ***factors of 16*** are: {1, 2, 4, 8, and 16}.

→***Prime number:*** an integer ***greater* than 1** that has only ***two*** different factors: number 1 and itself.

→***Composite number:*** an integer ***greater* than 1** that has ***<u>more</u> than*** two factors.

Example: ***2*** is a ***<u>prime number</u>***, since the only two factors are ***1*** and ***itself***

4, 8 and 16 are ***composite numbers***, since they all have ***more than two*** factors

Circle all the **prime** numbers in the chart below.

0	1	2	3	4	5	6	7	8	9
10	11	12	13	14	15	16	17	18	19
20	21	22	23	24	25	26	27	28	29
30	31	32	33	34	35	36	37	38	39
40	41	42	43	44	45	46	47	48	49

****Notice that the numbers 0 and 1 are **neither** prime nor composite****

- The number 1 has only ***one*** factor or divisor, not two or more.
- Zero has an ***infinite*** number of divisors, since zero can be divided evenly by any value and would still equal zero. Also, you cannot multiply two different non-zero values and have a product of zero.

1. Prime Factorization – Determining the Prime Factors of a Whole Number (p. 135)

The ***prime factorization*** of a number is the number written as the ***product of its prime factors***. Find ***two*** factors of the given value and write them as branches on a ***factor tree***.

Example: Use a factor tree to find the **prime factorization of 24** in three ways: start with 6x4, 2x12, 8x3). Six, four, twelve, and eight are **composite numbers** that can be factored further.

24 24 24

Note: No matter which two factors you started with, the prime factorization is the ***same.***

******There is only **ONE** correct **prime factorization** of a given value******

Therefore,

- the ***prime factors*** of *24* are ***2* and *3***

- the ***prime factorization*** of *24* (written as the ***product of its prime factors***)is: ***2 x 2 x 2 x 3***

- the ***prime factorization*** of *24* (written as the ***product of powers)*** is: $2^3 \times 3$

Example: Determine the ***prime factorization*** of *72*, using a factor tree. Write the prime factorization ***both*** as a *product of its prime factors*, and as a *product of powers*.

Example: Write the ***prime factorization*** of *168*. Write the prime factorization ***both*** as a *product of its prime factors*, and as a *product of powers*.

2. Determining the Greatest Common Factor (p. 136)

If ***two or more*** numbers have the ***same*** prime factor, it is called a ***common factor***. The **greatest common factor** is the greatest factor that 2 or more terms have in common.

Example: Determine the **prime** factors of *30* and *18*, using a factor tree. Highlight or circle the factors that appear in each prime factorization.

30 *18*

The ***common factors*** (the numbers you circled or highlighted) of *18* and *30* are *2* and *3*. The ***product*** of these common prime factors is called the ***greatest common factor***.

Therefore the ***GCF*** of *30* and *18* is *2 x 3*, or *6*. The GCF is the ***largest*** number that divides two or more numbers. If two numbers have ***no*** common prime factor, the *GCF* is *1*.

Example: Determine the ***greatest common factor*** of *60* and *28*

Example: Determine the ***greatest common factor*** of *12*, *30*, and *42*.

3. <u>Determining the Least Common Multiple (p. 137)</u>

<u>To determine the multiples of a number, multiply the number by the natural numbers</u> (1,2,3,4..) For example, the **<u>multiples of 26</u>** are 26, 52, 78, 104….

For 2 or more natural numbers, their **<u>Least Common Multiple</u>** is the smallest number that is a multiple of both or all the numbers. (*The **Least Common Multiple** is the number we use for the common denominator when adding or subtracting fractions). In other words, it is the **least** number that is <u>divisible by each number</u>.*

Example: Determine the **Least Common Multiple** of **18, 20, and 30**

(Draw the factor tree of each number in order to find the prime factorization of each number as a product of powers. Highlight or circle the greatest POWER of <u>each prime factor</u> in each list.) ***The least common multiple is the <u>product</u> of the <u>greatest power</u> of each prime factor.***

Try it! Determine the LCM of 28, 42, 63 (answer 252)

4. Problem Solving using GCF and LCM (p. 138)

a) What is the side length of the smallest square that could be tiled with rectangles that measure 8 in by 36 in? Assume the rectangles cannot be cut.

> The side length of the square must be a common multiple of 8 and 36.
>
> Write the prime factorization of each number (as a product of powers).
>
> Determine the LCM.

b) What is the side length of the largest square that could be used to tile a rectangle that measures 8 in by 36 in? Assume that the squares cannot be cut. Sketch the square and rectangles.

> The shorter side of the rectangle measures 8 in so the side length of the square must be a factor of 8.
>
> The longer side of the rectangle measures 36 in so the side length of the square must be a factor of 36.
>
> Write the prime factorization of each number.
>
> Determine the GCF.

Try "check your understanding" 4a,b p. 138

HOMEWORK: ________________________________ Due tomorrow.

Information about 3.1 can be found on page 134-139 in your textbook

3.2 Perfect Squares, Perfect Cubes, and Their Roots

p. 142

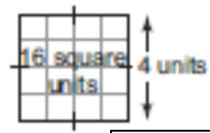
FOCUS Find square roots of perfect squares and cube roots of perfect cubes.

1 A **perfect square** is the square of a whole number.

For example, 16 is a perfect square because $16 = 4^2$

We say: 4 is the **square root** of 16.

We write: $\sqrt{16} = 4$



100 is a perfect square.

The prime factorization of 100 is:

$100 = 2 \times 2 \times 5 \times 5$, or $2^2 \times 5^2$

The prime factors occur in pairs.

This is true for any perfect square.

So, we can use prime factorization to find the square root of a perfect square.

A **perfect square** is the product of 2 <u>equal</u> whole numbers. The square root of a number is <u>one</u> of the equal numbers.

A perfect square can be represented as the **area of s square** with a whole number side length.

The **square root of the area** of the square is the side length of the square.

<u>Example: Write 1296 as the product of prime factors</u>.

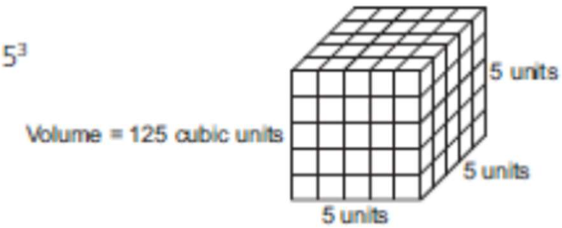
Group the factors in pairs. Rearrange the factors in two equal groups.

Since 1296 is the product of two equal whole numbers (______ x ______), its square root is one of these numbers. (In other words, 1296 is the area of the square and $\sqrt{1296}$, or _____ is the side length of the square.)

Try it: Determine the square root of 1764 with the prime factor method. (42)

2

A **perfect cube** is the cube of a whole number.
For example, 125 is a perfect cube because $125 = 5^3$
We say: 5 is the **cube root** of 125.
We write: $\sqrt[3]{125} = 5$



.

Example: Find $\sqrt[3]{1728}$, using prime factorization.
Group the factors in sets of 3. Rearrange the factors in three equal groups.

Since 1728 is the product of 3 equal factors, it can be represented by the volume of a cube. The side of the cube is equal to the cube root of 1728 (or the cube root of the volume). 1728 = _______x________x_________, so $\sqrt[3]{1728}$ = ___________________.

Try it: find the cube root of 2744, using prime factorization. (14)

3. A cube has a volume of 4913 in^3. What is the **surface area** of the cube?

(Remember if the volume is 4913, the side length is: _______________

Also remember the formula for the surface area of a cube is : ___________

(A cube has _____ faces. Each face is a ____________. The formula for area of a square is : _____________)

Homework: ______________________________

<u>Polynomials – Review of Vocabulary and Operations</u>

VOCABULARY

A ***term*** is a mathematical expression that can be a number, a variable, or the ***product*** of numbers and variables.

Examples of terms are: -5 **or** x **or** $5x$ **or** $-5xy$

A ***polynomial*** is a mathematical expression with one or more terms, formed by ***adding or subtracting*** terms.

Consider the polynomial: $\boldsymbol{-5x^2 + 4x - 2}$

-It is made up of ***three*** terms formed by an addition and a subtraction operation.

-The ***variable***, or unknown number, is represented by the letter, x.

-The exponents in the terms are given in ***descending order of power***, such as x^2 followed by x^1 followed by x^0

-When a term is written without a variable, such as -2, it is called a ***constant***, because it will always have the same value.

-When a term has both a number and a variable, such as $-5x^2$ and $4x$, the number is called the ***coefficient***, which tells you how many times to multiply the variable.

Polynomials can be named depending on the number of terms they have. Polynomial expressions with 1, 2, or 3 terms have special names:

Monomial – one term **Binomial** – 2 terms **Trinomial** – 3 terms

Polynomials with ***more than*** 3 terms are simply called polynomials.

Example: Complete the following chart.

Polynomial	# of terms	Name	Variables	Coefficients	Constants
$-x^2$					
$4y^3$					
$5x^2 - 1$					
$8r^2 - 4r + 2$					
$-6r^5 + 2r^3 - k - 10$					

OPERATIONS ON POLYNOMIALS - Review

Polynomials can be ***combined*** (added/subtracted) if they have ***like terms*** - two terms that have the ***same variable AND exponent***, and differ ***only*** by the ***numerical coefficient***.

Example: $\boldsymbol{17r^3t}$ and $\boldsymbol{-4r^3t}$ are **like terms**, because they have the same variables and exponents, $\boldsymbol{r^3}$ and $\boldsymbol{t^1}$, and differ only by their coefficient, ***17*** and ***-4***.

Addition: to add polynomials, you simply add the coefficients together, and keep the variables and exponents the same:

Example: $(3x - 3y) + (4y - 2x)$

$=3x - 3y + 4y - 2x$ (we remove the brackets without changing the terms)

$=3x - 2x - 3y + 4y$ (we group the x's and y's because they're like terms)

$= x + y$ (if the coefficient is 1, we don't write the 1)

Example: $(3x^2 - 5y) + (4y - 2x) + (-x^2 + 4x)$

Subtraction: to subtract polynomials, you **add** the opposite.

Example: $(4m^2 - 2m - 4) - (-3m^2 - 2m + 5)$

When there is a subtraction or **negative** sign <u>in front of a bracket,</u> in the next step we don't write the negative, we remove the bracket, and we change all the terms in the bracket to their **opposite sign**. The rest of the steps are the same as addition.

$(4m^2 - 2m - 4) - (-3m^2 - 2m + 5)$

$=4m^2 - 2m - 4 + 3m^2 + 2m - 5$

$=4m^2 + 3m^2 - 2m + 2m - 4 - 5$

$=7m^2 - 9$

We didn't change any of the terms in the first bracket because it had nothing or a positive in front of the bracket. We changed all the terms in the second bracket to their opposite signs (and didn't write the subtraction sign that was before the bracket).

Example: $(-2x^2 + 7) - (3x^2 + x - 5)$

OPERATIONS ON POLYNOMIALS – Review, p. 2

Multiplication: to multiply polynomials, you apply ***distribution*** and the ***exponent laws***. To begin, you multiply the coefficient by each term inside the brackets separately.

Example: $2(x + 5)$

→***Distribute*** the coefficient, 2, by multiplying it by the variable and the constant separately, so: $2(x) = 2x$ ***and*** $2(5) = 10$

Therefore: $2(x + 5) = 2x + 10$

Example: $-2(x + 5)$ = ______________________________

Example: $2x(x + 5x^2)$ = ______________________________

Division: to divide polynomials, you divide the coefficients and apply the ***exponent laws***.

Example: $8x^4y^3 \div -2x^3y$ or $\frac{8x^4y^3}{-2x^3y}$

→Begin with dividing the coefficients: $8 \div -2 = -4$

→Divide like terms separately. Remember the ***quotient law***: when dividing like terms, subtract the exponents. So, $x^4 \div x^3 = x^1$ or x ***and*** $y^3 \div y^1 = y^2$

Therefore: $8x^4y^3 \div -2x^3y = -4xy^2$

Example: Simplify: $6x^3y^2z \div 12xy^2$ *or* $\frac{6x^3y^2z}{12xy}$

3.3 Exploring prime factorization and GCF with variables (p. 150)

Quick review: Find the **prime factorization** of: *20*

Question: How could you apply what you know about **prime factorization** to the term:

$20x^2$

Quick review: Find the greatest common factor of:

20 *and* *36*

Question: How could you apply what you know about prime factorization to the term:

$20x^2 + 36x$

Question: How could you apply what you know about finding the **GCF** to the previous question?

Factoring Binomials – *Greatest* Common Factor

When a binomial is written as the ***product of its factors***, the binomial has been ***factored***. (To factor a polynomial, we write it as a product of its factors.) You can factor a polynomial using the ***greatest common factor*** method.

Example: Factor each term of the binomial: $4c^2 + 6c$

$4c^2 =$ ________ ***and*** $6c =$ ____________

Notice both $4c^2$ and $6c$ have a common _____ *and* _____, therefore the ***GCF is*** ______.

Therefore, $4c^2 + 6c =$ _________________________

(What do we multiply the GCF by to get $4c^2$ and $6c$? In other words, divide $4c^2$ by $2c$ and $6c$ by $2c$, remembering the quotient law where we subtract the exponents of the same variables.)

Example: Factor : $15n^2 + 6n$ $12s^2 + 3s$

Quick review: Go back to the first example, where you were asked to factor the binomial:

$$4c^2 + 6c$$

When the binomial is factored, the solution is: $2c\,(2c + 3)$

Now: Apply distribution to: $2c\,(2c + 3)$ *to check to make sure you have factored correctly.*

Factoring the binomial, $4c^2 + 6c$, resulted in the solution, $2c\,(2c + 3)$. When you apply distribution to $2c\,(2c + 3)$, it results in the binomial, $4c^2 + 6c$. This occurs because ***factoring and expanding (distributing) are inverse processes.***

Therefore, to verify that the solution to factoring is correct, you can use distributive multiplication.

<u>*In Arithmetic*</u>	<u>*In Algebra*</u>
Multiply factors to form a product	Expand an expression to form a product
$(4)(7) = 28$	$3\,(2 - 5a) = 6 - 15a$
Factor a number by writing it as a	Factor a polynomial by writing it as a product
Product of its factors	of its factors
$28 = (4)(7)$	$6 - 15a = 3\,(2 - 5a)$

Go back to the examples: $15n^2 + 6n$ and $12s^2 + 3s$, and apply distribution to verify your solution.

$15n^2 + 6n$ $12s^2 + 3s$

Recall the difference between $(-3)^2$ and -3^2. Write each as repeated multiplication.

This concept applies to factoring binomials with negative numbers.

Example: Factor the binomial: $-16t^2 - 24t$

$-16t^2$ $\quad\quad\quad\quad$ $-24t$

$-16t^2 = -1 \bullet 2 \bullet 2 \bullet 2 \bullet 2 \bullet t \bullet t$ ***and*** $-24t = -1 \bullet 2 \bullet 2 \bullet 2 \bullet 3 \bullet t$ ***The GCF is*** ____

It is important to note that the <u>*negative is -1*</u> *and* ***needs to be included in the GCF****. As a rule, when the* ***largest degree of a polynomial is a negative****,* <u>*factor it out*</u>*.*

$-16t^2 - 24t \div -8t = -8t\,(2t + 3)$

What would happen if you did ***not*** *include the negative, and the GCF was 8t?*

Example: Factor the binomial: $-12n^2 + 8n$

In this example, -1 is ***not*** *a common factor; however, because* <u>*the largest degree is a negative*</u>*, you still need to factor out the negative, and include it in the GCF.*

$-12n^2 + 8n \div -4n = -4n\,(3n - 2)$

This is done so that the ***leading (first) term in the brackets is*** <u>***positive***</u>*.*

Verify using distribution:

What would happen if you did ***not*** *include the negative, and the GCF was 4n?*

Example: Factor and verify using distribution:

$49e^2 - 14e$ $\quad\quad\quad$ $16h - 64h^2$ $\quad\quad\quad$ $12x^4 + 16x^2$.

Homework: ____

2. Common Factors of Trinomials in the form $ax^2 + bx + c$ (p. 153)

You can factor a trinomial by using the greatest common factor method.

Example: Factor the trinomial: $5c^2 - 10c + 5$

$5c^2$ $-10c$ $+5$

$5c^2 =$ ____________ $-10c =$ __________ ___ ***The GCF is*** ____

Divide the trinomial by the greatest common factor:

$(5c^2 - 10c + 5) \div 5 =$ _________________

Write the polynomial in factored form, which is the greatest common factor multiplied by the quotient:

____

Example: Factor the trinomial: $-18n^3 - 12n^2 + 6n$

Write the polynomial in factored form, which is the **greatest common factor** multiplied by the **quotient.**

When the **leading coefficient is a negative,** you can <u>factor out the negative</u> by multiplying the entire polynomial by -1 (each term becomes its opposite):

____

= ____________________________

3. Common Factors of Polynomials in More than One Variable (p. 154)

You can factor the polynomial by using the greatest common factor method.

Example: Factor the trinomial: $-12x^3y - 20xy^2 - 16x^2y^2$

Factor each term of the trinomial using a factor tree:

$-12x^3y$ $\quad\quad$ $- 20xy^2$ $\quad\quad$ $- 16x^2y^2$

The GCF is ________

Divide the trinomial by the greatest common factor:

$(-12x^3y - 20xy^2 - 16x^2y^2) \div -4xy =$ _______________________

Write the polynomial in factored form, which is the **greatest common factor multiplied** by the quotient:

____

Example: Factor the trinomial: $-20b^4c - 30b^3c^2 - 25bc$

HOMEWORK: ____________________________________

3.5 Multiplying Two Polynomials: The Distributive Property and FOIL (p. 161)

-When we have a term multiplied by a polynomial, we multiply that term by each of the terms in the polynomial.

Example: $2x(x + 5x^2 - 10x^3) =$ ____________________________

1. When multiplying a binomial by a binomial, we multiply each term of the first binomial by each term of the second binomial.

Example: $(x + 2)(x + 4) =$ ________________________________

= ____________________________ (simplify by combining like terms)

$\mathbf{(x + 2)(x + 4) = x^2 + 6x + 8}$ ← Notice that **8** is the PRODUCT of **4** and **2**
and **6** is the SUM of **4** and **2**.

An acronym that can be used to help you remember an order of distribution when multiplying a **binomial** by a **binomial** is **FOIL**, which stands for: ***First Outside Inside Last***

Example: $(4x - 1)(x + 3)$

Multiply the **first** term in each binomial

-Multiply the two **outside** terms in each binomial
-Multiply the two i**nside** terms in each binomial
-
-Multiply the **last** term in each binomial

Then simplify by combining like terms.

Example: simplify $(x + 5)^2$

Simplify $(x - 5)^2$

We call these perfect **square trinomials.**
(They are **trinomials** (3 terms), and the first and last terms are **perfect squares** (hence "**perfect square trinomial**"). The middle term is the **product** of the <u>square roots of the first and last terms, multiplied by 2</u>. The middle term can be **added** or **subtracted.)**

$(x + 5)(x - 5)$

*We call this a **difference of squares.*** (It is a **binomial** – 2 terms. The first and last terms are **perfect squares**. The terms are **subtracted** – hence "difference" of squares.)

Multiply the following polynomials.

a) $(4a^2b)(5a^2b^2)$

b) $\left(-\frac{3}{4}mn^2\right)(8m^3n^2)$

c) $5a(a+7)$

d) (-2g + 8) (7 – 3g)

e) $(y+3)(y+4)$

f) $(2a-5)^2$

Find the value when $m = -2$ and $n = 4$.

$m^2 + n(m-n)$

Solve the following equation.

$\frac{x}{2} - 3 = x + 1$

2. Factoring Trinomials of the Form *$ax^2 + bx + c$ ($a = 1$) – PRODUCT/SUM method* **(p. 163)**

Recall:

$(x + 2)(x + 4) = x^2 + 6x + 8$ ← Notice that **8** is the PRODUCT of **4** and **2**
and **6** is the SUM of **4** and **2**.

Since expanding (distributing) and factoring are inverse processes,
factoring the trinomial, $x^2 + 6x + 8$, should result in, $(x + 2)(x + 4)$.

When the leading coefficient (a) is equal to 1, ***always*** use the ***product sum method***. This method finds two factors whose ***product*** is equal to ***c***, and whose ***sum*** is equal to ***b***.

Example: Factor the trinomial: $x^2 + 10x + 24$

(The first step in factoring is ***always*** to see if there is a greatest common factor (GCF) (and if so, to factor using GCF method. However, when the **leading coefficient (a) is equal to 1**, there is **NO** greatest common factor.)

List the pairs of factors of 24: ____________________________________

Think: Which of these pairs of numbers could possibly add or subtract to equal 10?

(You are looking for factors whose product equals +24, and whose sum equals +10.)

Factors of 24 ***Sum of the Factors***

Because factoring and expanding are inverse processes, factoring a trinomial will result in binomial multiplication, or *(x + integer) (x + integer)*. Therefore, write the factors as the *integers* in each of the binomials:

$(x + 4)(x + 6)$

Example: Factor the trinomials **and verify using distribution (FOIL)**

a) $z^2 - 12z + 35$. B) $a^2 + 7a - 18$. C) $n^2 - 2n - 8$.

******NOTE: there are 2 rules that can help you decide which 2 factors to use******

1. *When **c is negative,** the two factors must have **different** signs (one positive and one negative). If **b is positive**, the greater factor is positive, while if **b is negative**, the greater factor is negative.*
2. *When **c is positive,** the two factors must have the **same signs** (2 positives **or** 2 negatives). When **c is positive**, b determines the sign of the two factors. If **b is positive**, the 2 factors are positive, and if b is negative, the two factors are negative.*

Factoring Trinomials in Ascending Order

As the first step when factoring, check to make sure the polynomial is written in descending order (and if it is not, rewrite the expression in descending order).

Example: Factor the trinomial: $-24 - 5a + a^2$

Hint: Rewrite the polynomial in descending order, and then factor.

Verify your answer:

Example: Factor the trinomial: $16 - 6x - x^2$

Verify your answer:

HOMEWORK: ____________________________________

4.Factoring Trinomials with a Common Factor <u>and</u> Binomial Factors (p. 165)

Review: Factor the trinomial: $-m^2 + 18m + 80$

When factoring a trinomial in the form $ax^2 + bx + c$, where the coefficient (a) is ***NOT*** equal to 1, the first step is ***always*** to see if there is a greatest common factor (GCF), and if so to factor using the GCF method.

Example: Factor the trinomial: $-4t^2 - 16t + 128$

Find the greatest common factor and write the sum as a product:

Since the leading coefficient is now equal to 1, use the* ***product sum method**

Divide the trinomial by the GCF, and multiply the GCF by the quotient.
Remember the -4 factored out in the first step:

Since factoring and expanding are inverse processes, check that the factors are correct by distributing them.

Example: Factor the trinomial: $10x^2 + 80x + 120$

Homework: ________________________________

3.6 Factoring Trinomials $ax^2 + bx + c$ ($a \neq 1$)

Expand and simplify the following: $(x - 4)(x + 2)$ and $(3d + 4)(4d + 2)$

How are they different?

When the leading coefficient (a) is ***not*** equal to 1, ***factoring takes a little bit more work. Here are a couple of methods to try:***

- ***Trial and error Method* (reverse foil)** ***Factor*** $ax^2 + bx + c$ where $a \neq 1$. (p. 172)
  (*Nail down your first and last terms and try different combinations to get your outer and inner terms to equal your middle term.)*
  WRITE LIGHTLY in PENCIL. HAVE ERASER HANDY! Or do on SCRAP PAPER.

Step 1: Place the factors of ax^2 in the first positions of the 2 sets of parentheses that represent the factors.

Step 2: Place 2 possible factors of **c** into the second positions of the parentheses (can leave out signs for a moment).

Step 3: Find the outer and inner products of the 2 sets of parentheses (O and I of FOIL)

Step 4: Keep trying different factors until the inner and outer products **add** or subtract to the middle term (bx).

Step 5: Decide on the signs of the second position terms so that they add/subtract to the middle term.

Examples: factor

1. $6x^2 - 19x + 15$
2. $15x^2 + 17x - 42$

- Box Method of factoring Trinomial $(a \neq 1)$ (p. 173)

For this method, initially we just write the number factors (not the variables)

Example: $4x^2 + 20x + 9$

Again, first we make sure there isn't a common factor that we could factor out. (___no___)

Think of factors of the first term and the last term. Write the factors vertically.

Now look at the products of the top and bottom numbers (diagonally). Add/subtract the products. Your goal is that the products will add or subtract to make the middle coefficient. (In this case to equal 20.)

You may end up trying all the combinations of the two sets of factors.. but you can stop if you find the pair that work.

4	3	4	9	2	3	2	9
1	3	1	1	2	3	2	1

As always, you can check your answer by multiplying (FOIL).`

Example: factor the trinomials and verify your answers. Remember the first step is to factor out the GCF, if possible.

$3s^2 - 13s - 10$ $\qquad$ $6x^2 - 21x + 9$

Homework: ________________________

3.7 Using the Distributive Property to Multiply **Polynomials p. 183**

Consider the example: $(2h + 5)(h^2 + 3h - 4)$. Using the distributive property, all terms in the first set of by all terms in the second set of brackets separately. Remember to **simplify by combining like terms**.

Example: Expand and simplify: $(2h + 5)(h^2 + 3h - 4)$

Distribute each term of the first bracket by each term of the second bracket Combine like terms or simplify

Examples: $(3b + 4)(b^2 - 2b - 7)$ $(-3f^2 + 3f - 2)(4f^2 - f - 6)$

2.Multiplying Polynomials in More than One Variable (p. 184)

Consider the example: $(x - y)(x^2 + 3xy)$. Using the distributive property, multiply both terms in the first set of brackets (binomial) by all both terms in the second set of brackets (binomial). Remember to simplify by combining like terms.

Example: $(x - y)(x^2 + 3xy)$

Examples: Expand and simplify:

a) $(4k - 3m)(4k + 3m)$ b) $(4k - 3m)^2$ c) $(4k + 3m)^2$

3. Simplifying Sums and Differences (p. 185)

Consider the example: $(2c - 3)(c + 5) + 3(c - 3)(-3c + 1)$. The problem has added order of operations, therefore, you must apply BEDMAS to solve.

Examples: Expand and simplify:

a) $(3x + y - 1)(2x - 4) - (3x + 2y)^2$ b) $(4m + 1)(3m - 2) + 2(2m - 1)(-3m + 4)$

Consider the example: $(3x + 4)(x - 5)(2x + 8)$. To solve this problem, you need to multiply the first two binomials, and then simplify. Next apply the distributive property to the product of the first two binomials and the third binomial.

Example: $(3x + 4)(x - 5)(2x + 8)$

Distribute each term of the first binomial by each term of the second binomial.

Distribute each term of the third binomial by each term of the product of the first two binomials.

*

HOMEWORK: ______________________

HOMEWORK: page 194 - 195 #11 - 12. Due tomorrow.

Factoring Special Polynomials

1. Factoring a Perfect Square Trinomial (p. 190) (see also p. 17 of notes)

Reminder: When factoring a trinomial in the form: $ax^2 + bx + c$, where the coefficient (a) is ***NOT*** equal to 1, the first step is ***always*** to find the greatest common factor.

Example: Factor the trinomials: $4x^2 + 12x + 9$ $\qquad\qquad$ $8x^2 + 40x + 50$

We call these **perfect square trinomials.**

(They are trinomials (3 terms), and the first and last terms are perfect squares (hence "perfect square trinomial"). The middle term is the product of the square roots of the first and last terms, multiplied by 2. The middle term can be added or subtracted.)

NOW when factoring:

1. Check for GCF (and factor out if possible)
2. Check to see if trinomial is a perfect square trinomial.
3. If not perfect square trinomial, carry on factoring as per usual.

Try these:

a) $4a^2 - 20a + 25$ $\quad$ b) $4 - 20x + 25x^2$ $\quad$ c) $4x^2 + 12x + 9$

Question: *Why can you only use the shortcut when either all three terms are positive, or when ONLY the second term is negative? Why does the shortcut not work when either the first or third term is a negative???*

3.8 2. Factoring Trinomials in Two Variables (p. 191)

When factoring a trinomial with two variables, you follow much the same process you would if you were factoring a trinomial with one variable.

Example: Factor the trinomial: $15m^2 + 7mn - 4n^2$

Always try to find the greatest common factor and write the sum as a product.

There is no GCF for $15m^2 + 7mn - 4n^2$

*****Check the leading coefficient to decide which method to use in factoring****

Since the leading coefficient is not equal to 1, one of the methods p. 22 or 23.

Add in all the variables at the end, once you have figured out the number values of the terms.

Verify your answer:

Example: Factor the trinomial: $x^2 - 3xy + 2y^2$

(Is there a GCF?)

*****Check the leading coefficient to decide which method to use in factoring****

Since the leading coefficient is equal to 1, use product sum.

Think of the pairs of factors of *2*, that add up to *– 3:*

Add in the variables at the end.

4. Factoring a Difference of Squares (p. 193) (and notes p. 17)

Expand the following, and collect like terms:

a) $(x + 3)(x + 3)$ b) $(x - 3)(x - 3)$

We call (a) a difference of squares. (It is a binomial – 2 terms. The first and last terms are perfect squares. The terms are subtracted – hence "difference" of squares.)

NOW when factoring:

1. Factor out the GCF if there is one.
2. If it's a binomial – is it a difference of squares?
3. If it's trinomial – is it a perfect square trinomial?
4. If not 2 or 3.. carrying on factoring as per usual.

Example: Factor:: $y^2 - 36$ $2x^2 - 2$

$100r^2 - 49s^2$ $3m^2 - 27n^2$

Sometimes, factoring differences of squares could occur more than once in each question.

Example: Factor $x^4 - 16$

NOTE: *The second polynomial after initial factoring could also be factored using difference of squares.*

Try these:

Example: $5x^4 - 80$

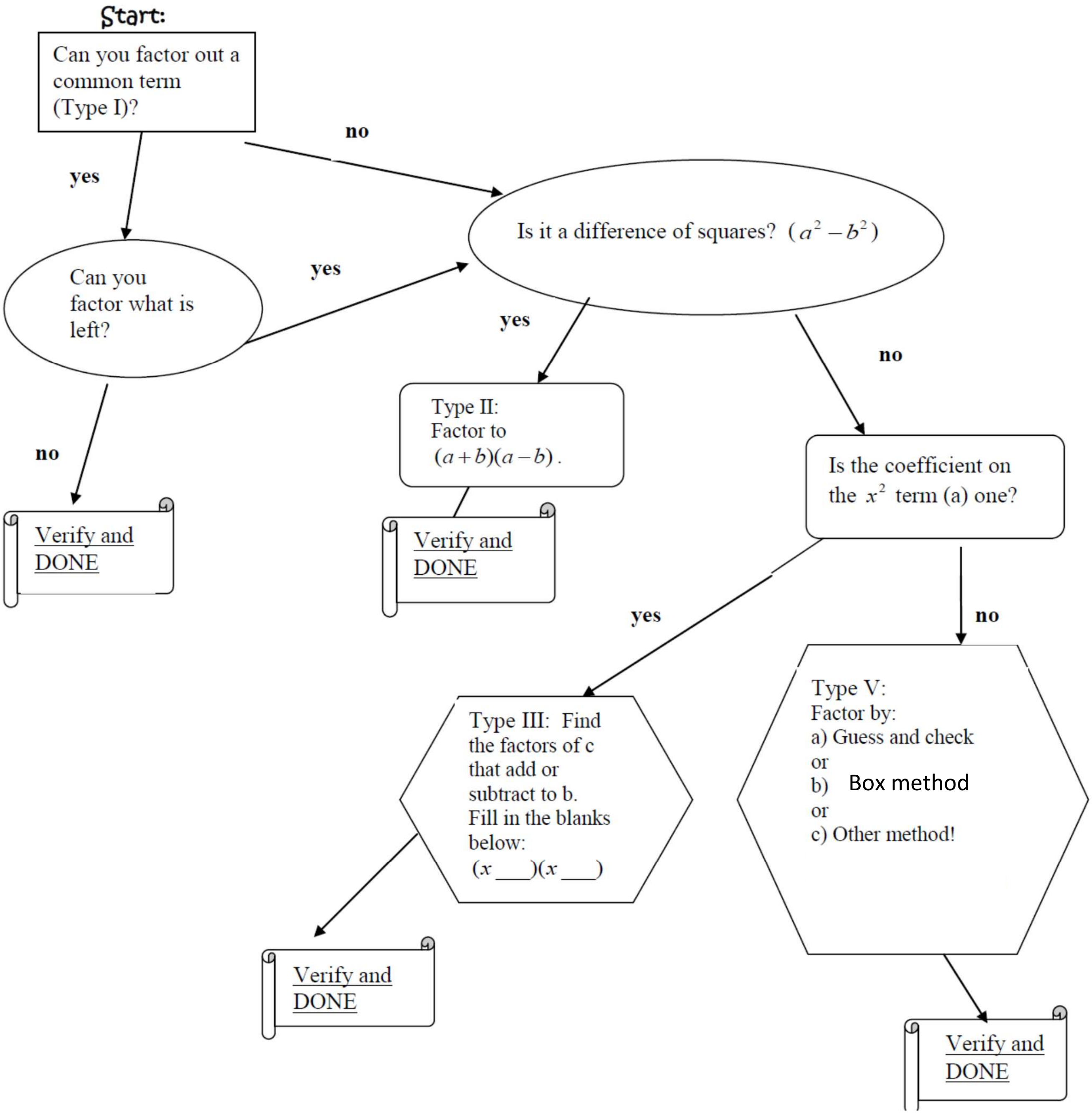
Example: $162v^4 - 2w^4$

ONCE YOU HAVE FACTORED.. ALWAYS CHECK TO SEE IF IT CAN BE FACTORED FURTHER.

HOMEWORK: ________________________________

Factoring Flow Chart

Quadratics: $ax^2 + bx + c$



Factoring- All Types!!!

Date________

Factor. Hint: take out the GCF

1) $5x^5 - 30x^2$

2) $-18v^2 - 9v$

3) $32x^2y^4 - 24xy$

4) $90a^9 + 27a^2b^3$

5) $20a^9b^5 + 4a^5b^4$

6) $2x^2 + 12x - 10$

7) $4ba^4 - 18b^3 - 8b$

8) $21xy^5 - 42x^2y + 49xy$

Factor. Hint: Starts with 1 (easy problems!)

9) $a^2 - 5a - 6$

10) $n^2 + n - 12$

11) $x^2 - 13x + 36$

12) $x^2 - 7x + 10$

Factor.

13) $10r^2 - 11r + 1$

14) $4k^2 + 8k - 21$

15) $4p^2 - 20p + 9$

16) $4m^2 - 39m - 10$

Factor. Hint: take out a commmon factor 1st!

17) $20x^2 + 82x + 80$

18) $20n^2 - 66n - 14$

19) $40x^2 + 44x - 24$

20) $12p^2 - 26p - 16$

Factor each completely. Hint: different of two squares

21) $4v^2 - 25$

22) $9b^2 - 4$

23) $x^2 - 16$

24) $9x^2 - 1$

Factor each completely. Hint: perfect squares

25) $9k^2 - 12k + 4$

26) $25a^2 + 20a + 4$

Directions: Factor each expression below completely. Show ALL Work!

8) $196 - 4y^2$	9) $2ax - 5x$	10) $24a^2b + 18abc$
11) $4x^2 - 24x - 28$	12) $49m^2 - 100n^2$	13) $20 + 9x + x^2$
14) $x^4 - 1$	15) $6x^2 - 6y^2$	16) $2x^3 - 8x$
17) $5x^2 + 20x + 20$	18) $5x^3 - 10x^2 - 15x$	19) $x^2 - x - 56$